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THE PROBLEM OF CONTROL UNDER CONDITIONS OF INCOMPLETE INFORMATION*

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The problem of controlling a differential system under conditions of incomplete information on the disturbance and phase states of the object, is considered. The problem is formalized as a problem of controlling an evolutionary system whose states are described by the information variable and which is solved by the method of programmed stochastic synthesis /1, 2/. The purpose this paper is to illustrate the application of the method to such problems.

1. Let us consider a controlled object whose state at a given instant t is defined by the n-dimensional phase vector $x[t] = \{x_i[t],\ i = 1, \ldots, n\}$. The object is acted upon by a control $u[t]$ and disturbance $v[t]$. The control is r-dimensional vector $u = \{u_j,\ j = 1, \ldots, r\}$ and the disturbance is an s-dimensional vector $v = \{v_j,\ j = 1, \ldots, s\}$. The quantities n, r and s can be any fixed natural numbers. We will treat the vectors as column vectors. The motion of the object $x[t_0[\cdot]\vartheta] = \{x[t],\ t_0 \leqslant t \leqslant \vartheta\}$ occurs within a given time interval $t_0 \leqslant t \leqslant \vartheta$ and is described by the differential equation

$$x^{\cdot} = A(t)x + B(t)u + C(t)v \tag{1.1}$$

where $A(t), B(t), C(t)$ are continuous matrix functions. The motion can begin from any position $\{t_0, x_0\}$. The Borel-measurable realizations of the control $u(t_0[\cdot]\vartheta] = \{u[t],\ t_0 < t \leqslant \vartheta\}$ and disturbance $v(t_0[\cdot]\vartheta] = \{v[t],\ t_0 < t \leqslant \vartheta\}$ are admissible. Every admissible realization is bounded in modulo over $t_0 < t \leqslant \vartheta$ by its own constant. The process $\{x[t_0[\cdot]\vartheta],\ u(t_0[\cdot]\vartheta],\ v(t_0[\cdot]\vartheta]\}$ is realized with the help of the quality index

$$\gamma_x = |x[\vartheta]| + \int_{t_0}^{\vartheta} [\Phi(t, u[t]) - \Psi(t, v[t])]\, dt \tag{1.2}$$

Here $\Phi(t, u)$ and $\Psi(t, u)$ are positive definite quadratic forms continuous in t, and $|x|$ is the Euclidean norm of the vector x.

In fact, the problem consists of constructing a control $u[t]$ resulting in the smallest possible value of γ_x.

We shall assume that the information concerning the states $x[t]$ is made available with a distortion of a certain n-dimensional vector variable $x^*[t]$. We also assume that the spent control $u[t]$ can be stored. Let us introduce the variable

$$q[t] = x^*[t] - \int_{t_0}^{t} X[t, \tau] B(\tau) u[\tau]\, d\tau \tag{1.3}$$

which we will call the information disturbance. Here $X[t, \tau]$ is the fundamental matrix of solutions for the homogeneous equation $x^{\cdot} = A(t)x$. We will choose the quantity

$$y[t] = \{q[t_0[\cdot]t],\ u(t_0[\cdot]t]\} \tag{1.4}$$

as the information variable. It consists of samples of the information disturbance $q[t_0[\cdot]t] = \{q[\tau],\ t_0 \leqslant \tau \leqslant t\}$ and the control $u(t_0[\cdot]t] = \{u[\tau],\ t_0 < \tau \leqslant t\}$ which have occurred up to the instant t. Let us postulate the piecewise continuous samples $q[t_0[\cdot]\vartheta]$.

We shall call the strategy $u(\cdot)$ the function

$$u(\cdot) = \{u(y[t], \varepsilon),\ t_0 \leqslant t < \vartheta,\ \varepsilon > 0\} \tag{1.5}$$

defined for every $t \in [t_0, \vartheta)$ for all possible values of $y[t]$ and the constants $\varepsilon > 0$. The control law U is defined for the interval $t_* \leqslant t \leqslant \vartheta$, $t_* \in [t_0, \vartheta)$ as a combination of three components

$$U = [u(\cdot), \varepsilon, \Delta\{t_i\}] \tag{1.6}$$

where $\Delta\{t_i\}$ denotes the partitioning of the segment $[t_*, \vartheta]$ by the points

$$t_i,\ i = 1, \ldots, k+1;\ t_1 = t_*,\ t_{i+1} > t_i,\ t_{k+1} = \vartheta$$

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The motion of the object $x[t_*[\cdot]\vartheta] = \{x[t], t_* \leqslant t \leqslant \vartheta\}$, generated by the control law U (1.6) from the position $\{t_*, x_*\}$ is defined as the solution of the stepwise differential equation

$$x^{\cdot}[t] = A(t)\,x[t] + B(t)\,u(y[t_i], \varepsilon) + C(t)\,v[t],\; t_i < t \leqslant t_{i+1},\; i = 1, \ldots, k \tag{1.7}$$

with initial conditions $x[t_*] = x_*$. This motion of the object x has the corresponding motion $y[t_*[\cdot]\vartheta]$ of the information y system whose states are described by the variable $y[t]$ (1.4). Here the components $q[t_0[\cdot]t_*]$ and $u(t_0[\cdot]t_*]$ of the initial state $y[t_*]$ can be one or the other, depending on the previous evolution of the system.

Let us now formalize the problem as the problem of controlling the y-system. The formalization can be constructed in one way or another, depending on the conditions of observation of the x-object. We shall assume that the distortion $x^*[t] - x[t]$ is estimated on the segment $[t_0, \vartheta]$ in the mean square. Then, using (1.2) we shall designate the following quality index for the y-system:

$$\gamma_y(y[\vartheta]) = \sup_{x[\cdot],\, v[\cdot]} [|x[\vartheta]| + \int_{t_0}^{\vartheta} [\Phi(t, u[t]) - \Psi(t, v[t]) - F(t, x^*[t] - x[t])]\,dt - F(t_0, x^*[t_0] - x[t_0])) \tag{1.8}$$

Here $F(t, x)$, $t \in [t_0, \vartheta]$ is a positive-definite quadratic form continuous in $t \in (t_0, \vartheta]$. The upper bound in (1.8) is computed over all possible motions $x[\cdot] = x[t_0[\cdot]\vartheta]$ and samples of the disturbance $v[\cdot] = v(t_0[\cdot]\vartheta]$.

We shall call the quantity

$$\rho(U; y[t_*]) = \sup_{y[\vartheta]} \gamma_y(y[\vartheta]) \tag{1.9}$$

the guaranteed result for the initial state $y[t_*]$ and control law U (1.6). Here the upper bound in computed over all possible realizations $y[\vartheta]$ continuing the initial state $y[t_*]$. Here the control $u[t]$ is formed, at $t > t_*$ according to the law U (1.6). We shall call the quantity

$$\rho(u(\cdot); y[t_*]) = \overline{\lim_{\varepsilon\to 0}}\, \lim_{\delta\to 0} \sup_{U_\delta} \rho(U_\delta; y[t_*]) \tag{1.10}$$

the guaranteed result for the strategy $u(\cdot)$ (1.5) and for the initial state $y[t_*]$. Here the upper bound is computed over all control laws $U = U^\delta$ (1.6) corresponding to the given strategy $u(\cdot)$, assigned to $\varepsilon > 0$, whose partions $\Delta_\delta\{t_i\}$ satisfy the condition $t_{i+1} - t_i \leqslant \delta$, $i = 1, \ldots, k$.

We shall call the strategy $u^0(\cdot)$, satisfying the relation

$$\rho(u^0(\cdot); y[t_*]) = \min_{u(\cdot)} \rho(u(\cdot); y[t_*]) \tag{1.11}$$

the optimal strategy for every possible initial state $y[t_*]$.

The problem is to construct the optimal strategy $u^0(\cdot)$. Using a less stringent formulation, we can state the problem concerning the optimal guaranteed result

$$\rho^0(y[t_*]) = \inf_{u(\cdot)} \rho(u(\cdot); y[t_*]) \tag{1.12}$$

2. Let us transform the index γ_y (1.8). By Cauchy's formula we have

$$x[t] = X[t, \tau]\,x[\tau] + \int_{\tau}^{t} X[t, \nu](B(\nu)\,u[\nu] + C(\nu)\,v[\nu])\,d\nu \tag{2.1}$$
$$t_0 \leqslant \tau \leqslant \vartheta, \quad t_0 \leqslant t \leqslant \vartheta$$

From (1.3), (1.8) and (2.1) we obtain

$$\begin{aligned}\gamma_y(y[\vartheta]) = \sup_{x[\vartheta],\, v[\cdot]} [\,|x[\vartheta]| + \int_{t_0}^{\vartheta} [\Phi(t, u[t]) - \Psi(t, v[t]) - \\ F(t, q[t] - X[t, \vartheta]\,x[\vartheta] - \int_{\vartheta}^{t} X[t, \nu]\,C(\nu)\,v[\nu]\,d\nu + \\ X[t, \vartheta]\int_{t_0}^{\vartheta} X[\vartheta, \nu]\,B(\nu)\,u[\nu]\,d\nu)]\,dt - F(t_0, q[t_0] - \\ X[t_0, \vartheta]\,x[\vartheta] - \int_{\vartheta}^{t_0} X[t_0, \nu]\,C(\nu)\,v[\nu]\,d\nu + X[t_0, \vartheta]\int_{t_0}^{\vartheta} X[\vartheta, \nu]\,B(\nu)\,u[\nu]\,d\nu)]\end{aligned} \tag{2.2}$$

Following the method of programmed stochastic synthesis /1, 2/, we shall formulate an auxiliary problem on the programmed extremum φ. Let $z[t,\omega]$, $\tau_0 \leqslant t \leqslant \vartheta$, $\tau_0 < t_0$ define a standard scalar process of Brownian motion /3/ defined in some probability space $\{\Omega, H, P\}$. Let the state $y[\tau_*]$, be realized at some instant $\tau_* \in [t_0, \vartheta]$, i.e. let a hindrance $q[t_0[\cdot]\tau_*] = \{q[t], t_0 \leqslant t \leqslant \tau_*\}$ and control $u(t_0[\cdot]\tau_*] = \{u[t], t_0 < t \leqslant \tau_*\}$ be realized.

We shall assign a partition $\Delta\{\tau_j\}$ for the segment $\tau_* \leqslant t \leqslant \vartheta$, where $j = 1, \ldots, k$, $\tau_1 = \tau_*$, $\tau_{j+1} > \tau_j$, $\tau_k = \vartheta$; k is a natural number. We will introduce an n-dimensional random vector quantity $l(\omega) = l[z_*[\tau_*[\cdot]\tau_k]]$, where the symbol $z[\tau_*[\cdot]\tau_i]$ denotes the sample

$$z[\tau_*[\cdot]\tau_i] = \{z[\tau_j,\omega] - z[\tau_{j-1},\omega], j = 1, \ldots, i\}, \omega \in \Omega$$

We shall also introduce an n-dimensional random vector quantity $w(\omega) = w[z[\tau_*[\cdot]\tau_k]]$ and any s-dimensional random measurable vector function $v(t,\omega) = v[t, z[\tau_*[\cdot]\tau_k]]$, $t_0 < t \leqslant \vartheta$, $\omega \in \Omega$. We shall call the non-anticipatory functions /3/

$$q(t,\omega) = q[t, z[\tau_*[\cdot]\tau_i]],\ u(t,\omega) = u[t, z[\tau_*[\cdot]\tau_i]]$$
$$\tau_i < t \leqslant \tau_{i+1},\ i = 1, \ldots, k,\ \omega \in \Omega$$

the stochastic programmes $q(\cdot)$ and $u(\cdot)$. We put

$$\varphi(y[\tau_*],\Delta) = \sup_{\|l(\cdot)\|\leqslant 1} \sup_{q(\cdot)} \inf_{u(\cdot)} \sup_{v(\cdot)} \sup_{w(\cdot)} M\Big\{l'(\omega)w(\omega) + \int_{t_0}^{\vartheta} [\Phi(t, u_*(t,\omega)) - \Psi(t, v(t,\omega)) - F(t, q_*(t,\omega) - X[t,\vartheta]w(\omega) - \int_{\vartheta}^{t} X[t,\nu]C(\nu)v(\nu,\omega)\,d\nu + X[t,\vartheta]\int_{t_0}^{\vartheta} X[\vartheta,\nu]B(\nu)u_*(\nu,\omega)\,d\nu)] \times dt - F(t_0, q_*[t_0] - X[t_0,\vartheta]w(\omega) - \int_{\vartheta}^{t_0} X[t_0,\nu]C(\nu)v(\nu,\omega)\,d\nu + X[t_0,\vartheta]\int_{t_0}^{\vartheta} X[\vartheta,\nu]B(\nu)u_*(\nu,\omega)\,d\nu)\Big\} \tag{2.3}$$

$$\|l(\cdot)\| = (M\{|l(\omega)|^2\})^{1/2}$$

The symbol $M\{\ldots\}$ denotes the expectation and the prime denotes transposition. The functions $u_*(\cdot)$ and $q_*(\cdot)$ are given by the equations

$$u_*(t,\omega) = \{u[t], t_0 < t \leqslant \tau_*;\ u(t,\omega), \tau_* < t \leqslant \vartheta\}$$
$$q_*(t,\omega) = \{q[t], t_0 \leqslant t \leqslant \tau_*;\ q(t,\omega), \tau_* < t \leqslant \vartheta\}$$

We shall call the quantity $\varphi(y[\tau_*],\Delta)$ the programmed extremum. Following the reasoning used to substantiate the method of programmed stochastic synthesis (see e.g. /2/), we can confirm the following assertion concerning the u-stability of the quantity φ.

Lemma 1. Let the state $y[\tau_*]$, $\tau_* \in [t_0,\vartheta]$ be realized, the partion $\Delta\{\tau_j\}$ assigned, the instant $\tau^* = \tau_2 \in (\tau_*, \vartheta]$ recorded, and let the realization of the disturbance $q^*(\tau_*[\cdot]\tau^*]$ be given. Then a realization of the control $u^*(\tau_*[\cdot]\tau^*)$ can be found such, that the inequality

$$\varphi(y[\tau^*],\Delta^*) \leqslant \varphi(y[\tau_*],\Delta)$$

holds, where the components $y[\tau^*]$ generate the components $y[\tau_*]$ by virtue of the samples $q^*(\tau_*[\cdot]\tau^*]$ and $u^*(\tau_*[\cdot]\tau^*]$, and the partition $\Delta^*\{\tau_j^*\}$ is connected with the partition $\Delta\{\tau_j\}$ by the condition $\tau_j^* = \tau_{j+1}$ with $j = 1, \ldots, k-1$. Let us write

$$\rho_*(y[\tau_*], c) = \sup_{\Delta} \varphi(y[\tau_*],\Delta) + c \tag{2.4}$$

where c is a scalar constant. Let $\lambda = 1 + \max[\|A(t)\|, t_0 \leqslant t \leqslant \vartheta]$ where $\|A(t)\| = \max|A(t)x|$ for $|x| = 1$.

For the assigned $\varepsilon > 0$ and the given state $y^*[\tau_*]$, we shall call the pair $\{y_*[\tau_*], c[\tau_*]\}$ satisfying the condition

$$\rho_*(y_*[\tau_*], c[\tau_*]) = \min_{y[\tau_*], g} \rho_*(y[\tau_*], g + \int_{t_0}^{\tau_*} [\Phi(t, u^*[t]) - \Phi(t, u[t])]\,dt)$$

under the constraint

$$q[t_0[\cdot]\tau_*] = q^*[t_0[\cdot]\tau_*]$$
$$\Big|\int_{t_0}^{\tau_*} X[\tau_*,\nu]B(\nu)(u[\nu] - u^*[\nu])\,d\nu\Big|^2 + c^2 \leqslant \varepsilon^2 \exp 4\lambda(t - t_0)$$

an associated pair. Here $q[t_0[\cdot]\tau_*]$, $u[t]$ are the components of the state $y[\tau_*]$; $q^*[t_0[\cdot]\tau_*]$,

$u^*[t]$ are the components of the state $y^*[\tau_*]$.

Let us determine the extremal strategy $u^{[e]}(\cdot)$ from the following condition of extremal translation from the state $y^*[\tau_*]$ to the associated pair $\{y_*[\tau_*], c[\tau_*]\}$:

$$l'(y^*[\tau_*], \varepsilon) B(\tau_*) u^{[e]}(y^*[\tau_*], \varepsilon) + s_{n+1}(y^*[\tau_*], \varepsilon) \Phi(\tau_*, u^{[e]}(y^*[\tau_*], \varepsilon)) = \min_u [l'(y^*[\tau_*], \varepsilon) B(\tau_*) u + s_{n+1}(y^*[\tau_*], \varepsilon) \Phi(\tau_*, u)]$$

Here the $(n+1)$-dimensional vector $s(y^*[\tau_*], \varepsilon) = \{l(y^*[\tau_*], \varepsilon), s_{n+1}(y^*[\tau_*], \varepsilon)\}$ is given by the equations

$$l(y^*[\tau_*], \varepsilon) = \int_{t_0}^{\tau_*} X[\tau_*, \nu] B(\nu)(u^*[\nu] - u_*[\nu])\, d\nu$$
$$s_{n+1}(y^*[\tau_*], \varepsilon) = c[\tau_*]$$

Lemma 2. Irrespective of the value of $\varepsilon > 0$, of the state $y^*[\tau_*]$ and of the number N, a $\delta > 0$ can be found such that whatever the disturbance

$$q(\tau_*[\cdot]\tau^*] = \{|q[t]| \leqslant N, \tau_* < t \leqslant \tau^*\}$$

generating the component $q^*[t_0[\cdot]\tau_*]$, the control

$$u(\tau_*[\cdot]\tau^*] = \{u[t] = u^{[e]}(y[\tau_*], \varepsilon), \tau_* < t \leqslant \tau^*\}$$

generating the component $u^*(t_0[\cdot]\tau_*]$ transfers the y-system to the state $y_*[\tau^*]$, whose associated pair $\{y_*[\tau^*], c[\tau^*]\}$ satisfies the condition

$$\rho_*(y_*[\tau^*], c[\tau^*]) \leqslant \rho_*(y_*[\tau_*], c[\tau_*])$$

as long as $\tau^* - \tau_* \leqslant \delta$.

Lemmas 1 and 2 together yield the inequality

$$\rho(u^{[e]}(\cdot); y[t_*]) \leqslant \rho_*(y[t_*], 0) \tag{2.5}$$

irrespective of the initial state $y[t_*]$. We further confirm that the following inequality holds for any strategy $u(\cdot)$:

$$\rho(u(\cdot); y[t_*]) \geqslant \rho_*(y[t_*], 0) \tag{2.6}$$

From (2.5) and (2.6) it follows that the extremal strategy $u^{[e]}(\cdot)$ is the optimal strategy $u^0(\cdot)$, and the following relation holds:

$$\rho(u^0(\cdot); y[t_*]) = \rho_*(y[t_*], 0) \tag{2.7}$$

The proof of the above assertions differs from those encountered in other analogous cases /2/ only in small details.

3. Sect.2 implies that to solve the initial problem of optimal strategy $u^0(\cdot)$ it is sufficient to solve the auxiliary problem of a programmed extremum $\varphi(y[\tau_*], \Delta)$ (2.3) for every possible state $y[\tau_*]$, $\tau_* \in [t_0, \vartheta)$.

Let us describe the plan for solving this auxiliary problem. We fix the random quantity $l(\cdot)$. Varying the random quantity $w(\cdot)$ and the random functions $v(\cdot)$, $u(\cdot)$, $q(\cdot)$ appearing in (2.3), we construct the equations expressing the necessary conditions of extremality for the quantity appearing in (2.3) under the symbol $\sup_{l(\cdot)}$. The equations are obtained by equating the corresponding variations to zero. We thereby obtain a system of linear integral equations for the conditional expectations

$$M\{l(\omega) \mid z[\tau_*[\cdot]\tau]\},\ M\{w(\omega) \mid z[\tau_*[\cdot]\tau]\}$$
$$M\{v(t, \omega) \mid z[\tau_*[\cdot]\tau\},\ M\{u(t, \omega) \mid z[\tau_*[\cdot]\tau]\},\ M\{q(t, \omega) \mid z[\tau_*[\cdot]\tau]\}$$

Let us take the random quantity $l(\cdot)$ in the form /3/

$$l(\omega) = l[\tau_*] + \sum_{j=1}^{k} a(\tau_j, \omega); \quad M\{a(\tau_j, \omega) \mid z[\tau_*[\cdot]\tau_{j-1}]\} = 0 \tag{3.1}$$

where $a(\tau_j, \omega)$ is a non-anticipatory function

$$a(\tau_j, \omega) = a[\tau_j, z[\tau_*[\cdot]\tau_j]] \tag{3.2}$$

Analysing the equations expressing the necessary conditions of extremality we find, that the extremal arguments $w(\cdot)$, $v(\cdot)$, $u(\cdot)$ and $q(\cdot)$, satisfying these equations, should be sought in the form of linear expansions in $q[t_0[\cdot]\tau_*]$, $u(t_0[\cdot]\tau_*]$, $l[\tau_*]$ and $a(\tau_j, \omega)$ where $q[\cdot]$ and $u[\cdot]$ are components of $y[\tau_*]$.

Thus the function $q(t, \omega)$ e.g. should be sought in the form

$$q(t,\omega)=\int_{t_0}^{\tau_*} Q_q(\nu)\,q[\nu]\,d\nu+Q_q^\circ q[t_0]+Q_l l[\tau_*]+\sum_{j=1}^{i} Q_a(t,\tau_j)\,a(\tau_j,\omega),\quad \tau_i<t\leqslant\tau_{i+1} \tag{3.3}$$

where $Q_q(\cdot)$, Q_q°, Q_l and $Q_a(\cdot)$ are the required matrices and matrix functions of the corresponding dimensions. The linear integral equations can be found for these matrices and matrix functions by substituting expansions of the form (3.3) into the right-hand side of (2.3) and varying the resulting expression over the matrices sought. The variation is carried out in the same order as the maximization and minimization operations in (2.3), from left to right. The resulting system of equations can be solved without fundamental difficulties, and reduces to routine though time-consuming procedures.

Substituting the expansions of the form (3.3) into (2.3), we arrive at the problem of computing the upper bound in the functions $l(\cdot)$ (3.1) of the known linear-quadratic functional $L(l(\cdot))=L[l[\tau_*],a(\cdot)]$. Thus the problem of computing the programme extremum $\varphi(y[\tau_*],\Delta)$ (2.3) reduces to that of finding the upper bound

$$\varphi(y[\tau_*],\Delta)=\sup_{l[\tau_*],a(\cdot)} L[l[\tau_*],a(\cdot)] \tag{3.4}$$

under the constraint

$$|l[\tau_*]|^2+M\left\{\sum_{j=1}^{k}|a(\tau_j,\omega)|^2\right\}\leqslant 1 \tag{3.5}$$

The auxiliary problem (3.4), (3.5) is identical, essentially, with the auxiliary problem dealt with in /2/ in connection with the solution, using the method of programmed stochastic synthesis, of the problem of game control when complete information is available on the phase states of the x-object. In /2/ it was shown that the problem considered here reduces to the case when the optimal partition Δ has the index $k\leqslant 2$. The present paper differs from /2/ in computing the parameters of the functional $L(\cdot)$.

Solving the problem (3.4), (3.5) by means of the method described in /2/, we find the quantity $\rho_*(y[\tau_*],c)$ which, according to (2.7), determines the minimal guaranteed result $\rho(u^\circ(\cdot),y[t_*])$. Constructing the extremal strategy $u^{[e]}(\cdot)$ on the basis of the quantity $\rho_*(y[\tau_*],c)$ (2.4) found, we obtain the optimal strategy $u^{[e]}(\cdot)$. The strategy $u^{[e]}(\cdot)$ in question can also be constructed using the numerical methods described in /2/. Thus we obtain for the problem of a minimum of the guaranteed result for the index $\gamma_y(y[\vartheta])$, a reasonably effective method for the numerical construction of the controls

$$u[t]=u^\circ(y[t_j],\varepsilon),\ t_i<t\leqslant t_{i+1},\ i=1,\ldots,k+1;\ t_1=t_0,\ t_1=t_0,\ t_{k+1}=\vartheta$$

carried out in the course of the actual control process.

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